When Less (Potential Demand) is More (Revenue): Asymmetric Bidding Capacities in Divisible Good Auctions*

Orly Sade
Jerusalem School of Business
Hebrew University of Jerusalem

Charles Schnitzlein
College of Business
University of Central Florida

Jaime F. Zender
Leeds School of Business
University of Colorado at Boulder

First Draft: May 5, 2004
Current Draft: May 10, 2005

* PRELIMINARY Comments are welcome, please, do not distribute or quote without the authors' permission. We thank Emmanuel Morales-Camargo, Ira Luria and Yelena Larkin for their excellent research assistance. We have benefited from comments by Yishay Yafeh, Eugene Kandel, Dan Levin, David Genesove, Eric Hughson, Steve Rock, and seminar participants at the University of Cincinnati, Hebrew University, the Federal Reserve Bank of Atlanta, Tel-Aviv University and Ben-Gurion University. Sade thanks the Krueger Center for Finance at the Hebrew University of Jerusalem for partial financial support.

When Less (Potential Demand) is More (Revenue): Asymmetric Bidding Capacities in Divisible Good Auctions

Abstract

We show that asymmetry in bidders' capacity constraints plays an important role in inhibiting collusion and promoting competitive outcomes in multi-unit common value auctions. This effect seems to be related to the increased difficulty of coordination when there are fundamental differences between bidders. The discriminatory auction is shown to be more susceptible to collusion than is the uniform-price auction and consequently asymmetry in capacity constraints plays a more important role in the discriminatory auction. These results suggest that the revenue maximizing auction format depends heavily on a variety of factors specific to particular auction settings.

1. Introduction

A variety of goods are being sold around the globe in multi-object or multi-unit (divisible good) auctions. Cellular phone licenses, energy, and mineral rights are all commonly sold via formalized auction procedures. There is also a widespread use of auctions in financial markets where IPOs, foreign exchange, and the treasury securities of many countries are all being sold via auction. Despite their widespread usage, fundamental questions concerning even the standard multi-unit formats (the discriminatory and the uniform-price auctions) remain unanswered. Notably, a general revenue comparison of the standard types of divisible good auctions has yet to be achieved. Given the size of many of these markets small improvements in the price per unit can have a tremendous impact on actual revenue.[1]

An important issue that is intimately tied to a revenue comparison of the standard multi-unit formats, particularly when the auction participants interact repeatedly, is the susceptibility of these auctions to collusion or collusive-seeming behavior in different settings.[2] Recent work in the theory of divisible good auctions shows that there exist non-cooperative equilibria under the uniform-price format that support collusive-seeming outcomes.[3] While the collusive seeming outcomes are not equilibria in the discriminatory auction, Friedman (1960) has argued that the discriminatory auction will be more susceptible to outright collusion and will suffer more from the winner's curse. Any theoretical comparison of the expected revenue generated in repeated uniform-price or discriminatory auctions will in part be determined by these forces.

[1] For example, Garbade and Ingber (2005) report that in 2003 the U.S Treasury auctioned $3.42 trillion of securities in a total of 202 auctions.
[2] Klemperer (2002) notes that: "The most important issues in auction design are the traditional concerns of competition policy - preventing collusive, predatory, and entry-deterring behavior."
[3] For example, see Wilson (1979), Back and Zender (1993), Ausubel and Cramton (1996) or Wang and Zender (2002) for theoretical evidence on strategic bidding in multi-unit auctions.

We use an experimental approach to examine a characteristic of auction markets that is predicted to affect the bidders' ability to collude; asymmetry in capacity constraints. An asymmetry is imposed on the bidders by preventing some of the participating bidders from bidding for the entire quantity of the good offered for sale. This investigation was motivated by the observation that in government securities auctions there are often a wide variety of players. Bidders in these auctions are generally both large and small financial institutions. In some countries individuals are also allowed to participate in the primary auctions. The different types of bidders have significantly different capacities to purchase the offered securities and this is predicted to impact their ability to collude. Additional motivation was provided by the Industrial Organization literature which shows that asymmetries between agents will often limit the ability of the agents in a given market to collude.[4]

The main results are that the introduction of asymmetry in the bidding capacities of the participants significantly reduced the amount of collusion and increased the revenue obtained by the seller. The increase in average revenue is shown to be the result of the reduction in collusive behavior. The discriminatory auction is more susceptible to collusion than is the uniform-price auction and the impact of the asymmetry is consequently greatest in the discriminatory auction. It is noteworthy that an increase in average revenue is generated despite the fact that the bidder asymmetry was obtained by reducing the total potential demand in the auction.

We use an experimental setting to examine this issue. The complexity of the equilibrium strategies in multi-unit auctions, the existence of multiple equilibria, and the repeated nature of many of the markets of interest make it difficult to analyze this issue theoretically. In addition,

---

[4] For a survey of the effects of capacity constraints on tacit collusion in oligopolistic industries see Rey (2003) pages 114-115.

the difficulty of obtaining rich data sets for these markets makes empirical work difficult and so an experimental study is a natural direction in which to turn.

Several papers have examined the issue of "collusive-seeming" strategies in the uniform-price auction. Goswami, Noe and Robello (1996) show experimentally that communication may increase bidders' tendency to play collusive or collusive-looking strategies. Back and Zender (2001) develop a model showing that when the seller retains the right to reduce the supply of the good offered at auction after observing the bids the bidders' ability to inhibit competition is limited. Kremer and Nyborg (2002 and 2003) show that, when prices, quantities and bids are discrete, the underpricing resulting from collusive-looking strategies can be made arbitrarily small by choosing a sufficiently small price tick size and a sufficiently large quantity multiple. They also show how one might improve revenues by modifying the allocation rule.

Recent experimental work has shown that the discriminatory auction is more susceptible to outright collusion on the part of bidders playing a repeated game. Sade, Schnitzlein, and Zender (2004) (SSZ hereafter) demonstrate that in their experimental setting bidders do not play the standard collusive-seeming strategies in the uniform-price auction rather the greatest threat to the seller's revenue is the use of outright collusion on the part of the bidders.

Pitchik and Schotter (1988) also investigate experimentally the role of budget constraints in sequential auctions under perfect information. In their setting the goods are heterogeneous and are sold one at a time sequentially. Their setting is different than ours in many ways since in our experiment the auctioneer sells multiple (26) homogenous units to several bidders at the same time. Yet, both papers find that the budget constraint affects bidding behavior. Different forces, however, appear to be at work across the two mechanisms. The intuition in the sequential auction mechanism for the effect of a capacity constraint on bidding behavior as described in

Benoit and Krishna (2001) is that when multiple objects are sequentially auctioned in the presence of budget constraints, it is advantageous for a bidder to bid aggressively on one object with a view to raising the price paid by his rival and depleting his budget so that the second object may then be obtained at a lower price. The intuition for the difference in bidding behavior in the divisible good auction is that the asymmetry in budget constraints makes it harder for bidders to collude since they find it difficult to agree on how to divide the surplus.

Armantier and Sbaï (2003) also study divisible good auctions with asymmetric bidders. They develop and test a model that includes agents who differ in terms of the precision of their private signals and their levels of risk aversion using data from French Treasury auctions. The complexity of their model requires that they use numerical methods to find a solution. They are also unable to consider the repeated nature of the game despite the fact that the auctions tend to involve largely the same players auction after auction.

This paper is organized as follows. Section 2 describes the theoretical foundations of our experiment. In section 3 we describe the experimental design, and the empirical results are presented in section 4. Section 5 contains concluding remarks. The appendix contains the written instructions that were provided to the participants.

2. Theory

In divisible good auctions bidders are able to submit multiple price quantity pairs (demand schedules) as bids. The submitted bid schedules are aggregated to form a downward sloping aggregate demand curve and the highest price at which aggregate demand equals the offered supply is the stop-out price. Winning bids are those submitted at or above the stop-out price. In a uniform-price auction the stop-out price is paid for all winning bids and in a discriminatory auction the bid price is paid for all winning bids.

In the present experiment we use an auction game in which $N = 5$ bidders compete for $Q = 26$ units of a good labeled a widget. In order to abstract from concerns over the winner's curse and concentrate on the strategic aspects of the auction, we ensure that it is common knowledge among the bidders that the after-market value of the widget (in Francs, the artificial currency of the experimental market) is 20Fr. We use a discrete price and quantity grid in which the "step size" in both dimensions is 1. In particular bids may be submitted as quantity orders at the 4 distinct prices contained in the set {17, 18, 19, 20}. Quantities must be for an integer number of widgets and the aggregate quantity demanded by a bidder is restricted to be either in the interval $[0, Q]$ or $[0, Q/2]$.[5] Each quantity order is an offer to purchase the specified number of units at a price equal to (equal to or below in the case of the uniform-price auction) the price at which the order is submitted.

Beginning with quantities submitted at a price of 20Fr, the "seller" aggregates all demands to create a downward sloping aggregate demand curve. The stop-out price is established as the highest price at which supply equals or exceeds the supply. Winning bids are those submitted at or above the stop-out price. All quantities demanded at prices strictly above the stop-out price are filled. Orders submitted at the realized stop-out price may be rationed.

Rationing is done in a *pro-rata* fashion. Specifically, the aggregate quantity demanded at the stop-out price is computed. For each bidder, the quantity demanded at the stop-out price is divided by the aggregate quantity demanded at the stop-out price to determine the proportion of

[5] We examine two different cases. In the "symmetric" case all bidders are allowed to bid for up to the entire quantity Q. In the "asymmetric" case 3 of the 5 bidders may bid for the entire quantity while 2 of the bidders may only bid for up to half the available supply. The asymmetry in the bidding capacity is the only difference between the current experimental design and the experimental design presented in SSZ (2004).

the rationed quantity that bidder is to receive. The rationed quantity is determined by subtracting the aggregate quantity demanded at all prices strictly above the stop-out price from the supply.

Given these characteristics we can describe Nash equilibria of the one-shot auction games for the symmetric capacity case in which all bidders may bid for the entire supply and for the asymmetric capacity case in which 2 of the five bidders are restricted from bidding for the entire supply and may only place bids for quantities up to $Q/2$. In both cases, the uniform-price mechanism supports multiple equilibria. In the symmetric capacity case, each of the possible prices may be the stop-out price in a symmetric Nash equilibrium. However, the only coalition proof equilibria are those for which the stop-out price is 17 (proposition 1). In the asymmetric capacity case, there are corresponding equilibria that are symmetric in the sense that all bidders with the same capacity (of the same type) play identical strategies (proposition 2). The discriminatory auction has a single equilibrium in undominated strategies in both the symmetric and the asymmetric cases. In this equilibrium, the stop-out price is 19 and all bidders submit demands for their maximum allowable quantity at this price (propositions 3 and 4).

Proposition 1 (symmetric capacities): *There exist symmetric Nash equilibria of the uniform-price auction that result in stop-out prices at any of the four possible price levels. (i) If all bidders submit demands for 3, 4, or 5 units at a price of 20, demand no units at 18, 21 units at a price of 17, and demand the balance of the 26 total units at 19, the equilibrium stop-out price will be 17. (ii) A stop-out price of 18 can be supported if all bidders submit demand curves with a total demand of 5 units at prices of 20 and 19, with at least 4 units demanded at 20, and 21 units demanded at 18. (iii) A stop-out price of 19 can be obtained in equilibrium if all bidders submit demands for 5 units at a price of 20 and demand for 21 units at 19. (iv) The competitive outcome is an equilibrium if all bidders submit demands for 26 units at a price of 20. In all of the symmetric equilibria each bidder will receive 5 and 1/5th units.*

Proposition 2 (asymmetric capacities): *There exist Nash equilibria of the uniform-price auction that result in stop-out prices at any of the four possible price levels. (i) If the total demand at the price of 20 is 15, 20, or 25 units, there is no demand at the price of 18, 79 units in total are demanded at the price of 17, and the balance of the allowable demand is made at the price 19, the equilibrium stop-out price will be 17. (ii) A stop-out price of 18 can be supported if the total demand is 25 units at the prices 20 and 19, with at least 20 units demanded at 20, and*

79 units are demanded at 18. (iii) A stop-out price of 19 can be obtained in equilibrium if the total demand is 25 units at the price 20 and a total of 79 units are demanded at 19. (iv) The competitive outcome is an equilibrium if all bidders submit the maximum allowable demand at a price of 20.

Proposition 3 (symmetric capacities): *The only Nash equilibrium in undominated strategies in a discriminatory auction has all bidders submitting demands for 26 units at a price of 19.*

Proposition 4 (asymmetric capacities): *The only Nash equilibrium in undominated strategies in a discriminatory auction has all bidders submitting the maximum allowable demands (either 26 or 13 units) at a price of 19.*

The theoretical results with respect to the equilibrium prices are exactly the same in the symmetric and asymmetric cases. However, the equilibrium allocations are different. In the symmetric case the allocations in the equilibria examined were, of course, all symmetric. Each bidder receives an allocation of 5.2 units in each auction. Given the asymmetry in the capacity constraint one expects to find that the players who have the ability to bid for more will receive a higher number of units. In the uniform-price auction the impact of the quantity restriction on the coalition proof Nash equilibria is marginal. The bidders who are allowed to submit bids for 26 units each have an equilibrium allocation of 5.266 units while those who are restricted to a total demand of 13 units have an equilibrium allocation of 5.101 units. This occurs because only one unit of the good is rationed amongst the 5 bidders. In the discriminatory auction in the asymmetric case, because all the participants submit their entire allowable demand at the price of 19, the entire supply of 26 units is rationed. Bidders who are able to submit demands for 26 units will, therefore, get twice the allocation of those who can submit for only 13 units (6.5 and 3.25 respectively).

In summary, the theoretical predictions from the Nash equilibria of the one-shot game are:

1. Average revenue to the seller should be higher under the discriminatory mechanism than it is for the uniform-price auction for both the symmetric and the asymmetric cases.

2. The average revenue to the seller (for both the uniform-price and discriminatory auctions) in the asymmetric capacity constraint treatment should be the same as that obtained in the symmetric capacity treatment.

Further, given the findings of SSZ (2004) which found the discriminatory auction format was more susceptible to collusion, if the asymmetry in capacity constraints is effective in limiting collusion we expect:

3. There should be less collusion in the asymmetric capacity treatments than in the symmetric capacity treatments.
4. Asymmetric bidding capacities should have a larger impact, in terms of restricting collusion and raising revenue, in the discriminatory auctions than in the uniform-price auctions.

3. Experimental Design

3.1 Auction Rules

Our auctions rules are based on SSZ (2004) with one major difference – we introduce asymmetry in bidding capacities.[6] Specifically, in this setting, in each auction, subjects bid for units of a good that we call widgets. As discussed above, there were 26 widgets available for sale in each auction. All monetary values are denominated in an experimental currency referred to as Francs (Fr). The resale value of each widget auction was 20Fr for all subjects, and this was common knowledge at the start of bidding. Subjects submitted bid schedules at computer terminals. Three of the subjects were permitted to bid for at most 26 units in total (the total supply) at the permissible prices 17Fr, 18Fr, 19Fr, and 20Fr while two of the subjects were permitted to bid for at most 13

[6] Here, we only consider mechanisms with fixed supply and do not consider the reducible supply case examined in SSZ (2004).

units. Although it was common knowledge that bidding capacity was asymmetric across the players, the exact distribution of bidding capacity was not necessarily common knowledge, each participant had the right but not the obligation to truthfully reveal his/her bidding capacity constraint.

Once all the bid schedules are submitted, the computer assigns widgets to subjects, allocating supply to the highest bids. When necessary demand at the stop-out price was rationed on a pro-rata basis as discussed above. In the uniform-price auctions, all the subjects pay the same price (the stop-out price) for each widget allocated and their payoff from each auction equals the difference between the resale value for each widget (Fr.20) and the stop-out price times the number of widgets allocated. Under the discriminatory auction, the computation of each subject's payoff is similar except that each unit allocated is sold at the bid price for that unit.

3.2 Experimental Methodology

Each experimental session consisted of 5 subjects and each cohort of 5 subjects was involved in a single experimental treatment. We employed both students from Israel and from the US in this study.[7] The students were undergraduate, MBA, MA and MIS students. All had had at least one course in finance, and courses in statistics and economics.

Table 1 lists the information pertaining to each experimental session. Eight repetitions of the uniform-price (four in Israel, and four in the US) and eight repetitions of the discriminatory (four in Israel and four in the US) treatments were conducted. There were at least 14 auctions conducted in each experimental session with the exact number chosen randomly. In order to control for experience effects, we analyze only the first 14 auctions in each experimental session.

[7] The experiments in the US were conducted at the University of Central Florida and the experiments in Israel were conducted at the Hebrew University of Jerusalem. Two different locations were chosen as a robustness test to check that cultural differences are not driving our results. As a further robustness check we also utilize data from SSZ (2004), this data was gathered at the University of Arizona.

At the start of each experimental session, subjects were seated in a conference room and given written instructions (in English in the U.S and in Hebrew in Israel). The instructions explained the auction rules, the basis on which cash payments would be made, and included images that introduced the subjects to the software used to conduct the experiment. The instructions were read aloud, and subjects were then given the opportunity to ask clarifying questions. The student subjects were then given a quiz to ensure their understanding of the bidding and allocation rules. (A copy of the written instructions in English, some sample computer screens, and the quiz are included in the appendix.)

Subjects were allowed to discuss strategies and outcomes with each other before, during, and after each auction. The layout of the computer lab, however, prevented each subject from seeing the screen of any other subject, and subjects were informed that this would be counter to the auction rules. Therefore, while communication was open, actual bidding behavior remained private knowledge. After the final auction in each session each subject's screen automatically reverted to a blank screen (to maintain the privacy of bidding behavior as subjects left the lab) and student subjects were paid individually in a side room. Payments to student subjects averaged $20.

The auctions were conducted with custom designed software. In addition to allowing the entry of bids, the software graphed individual demand curves in real time as each subject initiated the bid submission process. The aggregate demand schedule, stop-out price, and allocations for each round were calculated by the software at the completion of each auction. After each round each bidder was provided with information on the number of total units demanded at each price. In addition, the interface provided historical information pertaining to each subject's previously submitted demand functions matched with their allocations, profit, and percentage of available supply received for each completed auction. Each experimental session lasted approximately one hour.

4. Experimental Results

In SSZ (2004), two versions of a uniform-price auction mechanism were found to be less susceptible to collusion than the discriminatory-price mechanism. Our primary interest therefore is in how asymmetric bidding capacities affect the competitive dynamic and the relative performance of the uniform-price and the discriminatory mechanisms. Before taking up these issues we first consider some characteristics of the bidding behavior.

4.1 Bidding Basics

Proposition 2 notes that bidding in the discriminatory auction for any quantity at a price of 20Fr is a weakly dominated strategy in a one-shot game. In SSZ (2004), this dominated strategy was rarely played: in only 3 of the 70 discriminatory auctions did any bidder submit orders at a price of 20Fr.[8] It is also true that all such bids came from the same bidder. Thus only 1 of the 25 bidders that participated in the discriminatory auctions submitted a bid that was weakly dominated in this way. In our sessions with symmetric capacities, which replicate SSZ (2004), a similar result obtains: bidders only played this dominated strategy in 5 of the 112 discriminatory auctions. Only 3 of the 40 bidders that participated submitted such bids.

Asymmetric bidding capacities increase the propensity for bidders to submit orders at a price of 20Fr. In 5 of the 8 discriminatory sessions there were multiple auctions in which at least

[8] SSZ (2004) used both students and professionals as subjects. Since statistical analysis demonstrated clear differences in behavior between student and professional subjects when we compare outcomes from these new experimental markets (asymmetric bidding capacities) with those in SSZ (2004) (symmetric bidding capacities) we only utilize the sessions with student subjects (these comprised 71% of the sessions).

one bidder employed this strategy. In total, 11 of the 40 bidders employed this strategy at least once. Considering each bidder to be an independent observation, bidders are more likely to employ this strategy under asymmetric bidding capacities ($p=0.04$). In total, at least one bidder had 20Fr as part of his/her demand curve in 25 of 112 auctions. This ratio is significantly higher than in the symmetric bidding capacities setting ($p<0.01$).

Although bids at 20 are weakly dominated in the one-shot game they can be part of a retaliation/punishment strategy in our multiple auction setting. Determining the motive for such a bid is difficult; however evidence on whether they are due to "mistakes" can be gained by examining when they occurred in the session. Bids at 20Fr. in the first auction would seem most likely due to a misunderstanding of the auction rules, rather than due to a retaliation punishment strategy, since no auction information is received until all bidders have submitted their demands. After the first auction, bidders receive clear feedback that would indicate that bids at 20Fr. are never profitable. In fact, there were only 2 bids at 20Fr. (for a single unit) in the first auction of any of the 8 sessions. Weighting the timing of such bids by the quantity demanded at 20Fr. the average occurrence of bids at 20Fr. was between the 10th and 11th auction in each session. Furthermore in no case did this occur after the bidders had successfully colluded. This evidence is consistent with these bids being part of a strategy to induce cooperation in future auctions. The increased use of this type of bid is consistent with there being a more difficult coordination problem in the presence of asymmetric bidding capacities.

If the occurrence of bids at 20Fr is an indication of punishment strategies being played it is interesting to see if the use of such strategies differs as a function of the capacity constraint. 6 (of 16 or 37.5%) capacity constrained bidders submit bids at 20Fr in the asymmetric discriminatory auctions while only 5 (of 24 or 20.8%) of the unconstrained bidders submitted such bids. This difference, however, is not statistically significant ($p = 0.30$ using a Fisher exact test). Accounting for volume by comparing the proportion of total potential volume bid at 20Fr across the two groups we see that 3.3% of total demand is submitted at a price of 20Fr by capacity constrained bidders while only 1.6% of total demand is submitted at a price of 20Fr by the unconstrained bidders. Viewing each auction as an independent observation this difference is significant ($p < 0.01$) however this ignores the possible dependence between auction outcomes in a session. Using the most conservative correction for this problem, using each session as an independent observation, this difference is not significant ($p = 0.40$). Overall the evidence suggests that both types of bidders pursue punishment strategies with equal vigilance.

The theoretical results also identify bidding for fewer units than a bidder's capacity in any auction as a weakly dominated strategy. In SSZ (2004) this occurred in only 11 of the 140 (7.9%) comparable auctions. The use of this strategy tended to be isolated in the sense that in none of the 11 auctions with aggregate demand less than 130 units did more than one bidder bid for less than 26 units. In our symmetric capacity markets, this strategy was employed in 12 of the 224 (5.4%) comparable auctions, and in only one of these 12 auctions did more than one bidder bid for less than 26 units. On average, bidders bid for 99.9% of their capacity.

In sharp contrast, bidding below capacity is much more frequent when bidders have asymmetric bidding capacities. Bidding below capacity occurred in 10 of the 16 sessions and 79 of the 224 auctions (35.3%). In 66 of these 79 auctions at least two bidders followed this strategy. These differences relative to the sessions with symmetric bidding capacities are highly significant ($p<0.01$). Bidding less than capacity seems to play an important role in the development of collusive strategies when bidders have asymmetric capacities.

A final point pertaining to bids below capacity concerns the propensity to bid below capacity as a function of the capacity constraint. Under the discriminatory mechanism, bidders with a capacity constraint of 13 on average bid 88.4% of their capacity, while bidders with a constraint of 26 on average bid 76.4% of capacity. Under the uniform-price mechanism a similar result obtains with average bids of 92.6% and 85.1% of their capacities respectively. Weighting each auction equally (n=112), both differences are significant (p<0.01).

In sum, our evidence indicates that bidders behave very differently in the symmetric and asymmetric capacity environments. In the asymmetric setting bidders behave differently as a function of their capacity constraint. We next ask whether these differences in behavior relate to the different levels of collusion we observe and whether they result in different levels of revenue.

4.2 Collusion and Nash equilibrium

The experimental setting is one in which there are sufficient competitors to expect the competitive outcome, although by permitting open communication between the bidders, the barriers to cooperation and coordination are low. SSZ (2204) found substantial collusion, and that the discriminatory mechanism was more susceptible to collusion than was the uniform-price mechanism. In this section we examine the role that asymmetry in bidding capacities plays in inhibiting collusion.

We start by investigating how asymmetry in bidding capacities affects the likelihood an entire session will be collusive. SSZ (2004) defined a "perfectly collusive outcome" to be an auction outcome in which each bidder submits his/her entire demand of 26 units at the lowest permissible price (17) and a perfectly collusive session as one in which all auctions are perfectly collusive. This implies revenue to the auctioneer of 442 in each perfectly collusive auction. Under the uniform-price mechanism, perfect collusion is distinguished from the coalition-proof

Nash equilibrium that implies the same revenue by the incentive each bidder has to defect (in a one-shot game) from this agreement. Adjusting for the capacity constraints, we use a similar definition of collusion here.

In SSZ (2004), two of the five sessions conducted with students under both the discriminatory and uniform-price mechanisms are perfectly collusive sessions. In our sessions that replicate SSZ (2004) (symmetric bidding capacities), one of eight sessions is collusive under the uniform-price mechanism and three of eight are collusive under the discriminatory mechanism. With asymmetric capacity constraints, 1 of 8 sessions is perfectly collusive in both the uniform-price and discriminatory auctions. In total (including the markets from SSZ (2004)) 8 of 26 (30.8%) sessions are collusive when capacity constraints are symmetric.[9] With asymmetric capacity constraints the number of collusive sessions falls to 2 of 16 (12.5%). Thus, consistent with prediction 3, asymmetry in bidding capacities seems to play a role in inhibiting collusion, however, (given the relatively small number of sessions) the differences do not reach statistical significance (p=0.27).

Because the discriminatory auction has been shown to be more susceptible to collusion asymmetry in bidding capacity is predicted to have a more pronounced effect on it than it will have on the uniform-price auction. Therefore, we now examine the impact of capacity constraints controlling for the type of auction. As stated above, for the discriminatory auction with symmetric bidding capacities 37.5% of the sessions are perfectly collusive while with asymmetric bidding capacities only 12.5% of the sessions result in the perfectly collusive outcome for all 14 auctions (the p-value for this difference is 0.285). When we examine the number of perfectly collusive auctions under the discriminatory mechanism we find that with symmetric bidding capacities, 56 of 112 auctions (50%) were collusive while with asymmetric

[9] Excluding the markets from SSZ, 25% of the sessions are collusive

bidding capacities a significantly smaller number (35 of 112 or 35.3%) of auctions were collusive ($p<0.01$, treating each auction as an independent observation).

It is also interesting to consider the behavior of the bidders in the final auction of each session as an indication of the steady state outcomes to which the bidders converge. In the symmetric capacity version of the discriminatory auction, the participants converged solely to the Nash equilibrium (3 of 8 sessions) or to the perfectly collusive outcome (5 of 8 sessions, see table 2 sections C and D). When we introduced asymmetric bidding capacities to the discriminatory auction the participants converged to the perfectly collusive outcome less frequently (3 of 8 sessions) and converged to the Nash outcome in only a single session. As Table 3 reports, this difference in outcomes is significant ($p = 0.08$).

We can also examine differences in how sustainable the collusive outcome was across the different capacity treatments. Conditional on the perfectly collusive outcome in any auction the probability that the collusive outcome was realized in the subsequent auction of the same session was 96.1% when bidders had symmetric bidding capacities. When bidders had asymmetric capacity constraints this conditional probability was 87.5%. This difference is significant at the 10% level in a one-tailed proportional test.

Similarly, conditional on revenue equal to 494 in any auction (the symmetric Nash equilibrium of the discriminatory auction) the probability that revenue of 494 was realized in the subsequent auction across the different capacity treatments is as follows. With asymmetric bidding capacities this probability is 45% while with symmetric bidding capacities it is a significantly higher 77% ($p=0.02$). It is therefore true that asymmetric bidding capacities made it less likely that either the collusive outcome or the Nash equilibrium was sustained in successive auctions relative to the symmetric treatments.

Finally, consistent with prediction 4, note that none of the differences reported above for the discriminatory auctions were significant when we examined the uniform-price auctions with symmetric versus asymmetric bidding capacities (see Table 3). These differences in the case of the discriminatory auction suggest that asymmetric bidding capacities make it harder for bidders to reach and sustain collusive outcomes. We conjecture that a driving force of this result is the difficulty the subjects face in reaching agreement on how to divide the surplus that results from collusive strategies.

4.3 Clearing Prices and Revenue

In this section we analyze the influence of the auction mechanism and asymmetry in capacity constraints on the auctioneer's revenue. Mean clearing prices and revenue by session are reported in Table 1. With symmetric capacity constraints, average revenue is 466.6 under the discriminatory price mechanism and 472.4 under the uniform price mechanism.[10] With asymmetric capacity constraints, average revenue is 473.5 under the discriminatory mechanism and 473.6 under the uniform-price mechanism. Consistent with prediction 4 the impact of asymmetric capacity constraints is greater in the discriminatory auctions.

Figure 1 shows patterns in revenue as the subjects gain experience in a session. For each time series, the first point represents the average for all 14 auctions under each mechanism, the second point is the average for auctions 2 through 14, the third is the average for auctions 3 through 14, and so on. It is interesting to note that with asymmetric bidding capacities, revenue increases toward the end of the session under both auction mechanisms, while the reverse effect obtains with symmetric bidding capacities. This effect occurs in each of the groups of sessions: UCF (symmetric bidding capacities), UCF (asymmetric bidding capacities), Hebrew University

[10] In SSZ (2004), taking into account only the sessions with students as participants, average revenue was 461.6 under the discriminatory mechanism and 462.1 under the uniform-price mechanism.

(symmetric bidding capacities), Hebrew University (asymmetric bidding capacities), and University of Arizona (symmetric bidding capacities). Figure 1 demonstrates that the differences in behavior introduced by asymmetric bidding capacities are not driven by experience effects.

We next examine the relative importance of auction design and asymmetry in capacity constraints in determining revenue by estimating the following (OLS) regression:

$$\text{(1) REVENUE} = b_1 + b_2\text{UF} + b_3\text{ASYMMETRY} + \varepsilon$$

The variable definitions are as follows. REVENUE is revenue from the last auction in each session.[11] UF is an indicator variable that takes on a value of 1 in uniform-price sessions and 0 otherwise. ASYMMETRY is an indicator variable that takes on the value of 1 in the sessions with asymmetric capacity constraints and zero otherwise. The indicator for the discriminatory mechanism is suppressed; the mean for the discriminatory sessions is captured by the intercept after controlling for ASYMMETRY, and the estimated coefficient for UF represents the difference between these session means and the mean for the discriminatory sessions. We estimate the model with the 10 sessions from SSZ (2004) and the 32 sessions with symmetric and asymmetric capacity constraints conducted for this study (N=42). The adjusted R^2 is 3.6%.

The outcome of this regression is as follows:

$$\text{REVENUE} = \underset{(70.01)}{\underset{459.7}{b_1}} + \underset{(-0.17))}{\underset{-1.38}{b_2\text{UF}}} + \underset{(1.88)}{\underset{+15.69}{b_3\text{ASYMMETRY}}}$$

The coefficient on the indicator for asymmetry in capacity constraints is positive and significant at the 10% level (p=0.07). The revenue difference between the uniform-price and the discriminatory mechanism is not significant (p=0.87). These estimates indicate that asymmetry in bidding capacities increases expected revenue in both types of auctions. With the results in

[11] The last auction is used because experience has differential effects on competition as a function of capacity constraints. The last auction is most likely to be the closest to steady state outcomes.

section 4.2 on collusion, this suggests that impact the on revenue of asymmetry in bidding capacities is driven by the role they play in facilitating competitive outcomes.[12] The adjusted R^2 indicates substantial variation in revenue not captured by this simple model. This is due to the difficulties inherent in reaching and maintaining a collusive agreement, and the possible role of cohort effects.

In this experiment bidders cannot bid above 20Fr per unit or below 17Fr per unit so there is a minimum and maximum revenue is imposed by the experimental design. Therefore, we also estimated a Tobit version of equation (1). The results are qualitatively the same. The coefficient on the indicator for asymmetry in capacity constraints is positive and significant at the 10% level (p=0.06) while the revenue difference between the uniform-price and the discriminatory mechanism is not significant (p=0.66).

To further investigate the role of asymmetry in bidding capacities we perform an ANOVA analysis for the differences in conditional average revenue (again measuring revenue as the revenue in the final auction of each session) controlling for the following conditions: auction type (discriminatory or uniform-price), capacity constraints (symmetric or asymmetric), and the nature of the outcome (perfectly collusive or not). In all collusive outcomes the revenue is, by definition, 442 so these cases are not separately reported. For the discriminatory auctions with symmetric bidding capacities which were not collusive average revenue was 494. With asymmetric bidding capacities the discriminatory auctions that were not collusive the average revenue was 499.8. This difference is not significant (p = 0.49). Thus controlling for the absence of collusion there is not a significant difference in revenue across the different capacity constraints in the discriminatory auction. This is consistent with prediction 2. Consistent with

[12] Excluding the sessions from SSZ (2004) increases the p-value on the asymmetry coefficient to 0.26, and reduces the p-value on the UF mechanism coefficient to 0.86.

prediction 3, in the discriminatory auctions the results of the regression in equation (1) and the ANOVA analysis demonstrate that the impact of asymmetric capacity constraints on revenue derives from its role in limiting collusive behavior.

For the uniform-price auctions with symmetric bidding capacities that were not collusive the average revenue was 475.4. The uniform-price auctions with asymmetric bidding capacities that were not collusive had an average revenue of 488.8.[13] This difference is significantly different from zero ($p = 0.08$). Thus controlling for collusion, asymmetric bidding capacities has a significant impact on revenue in the uniform-price auction. This is contrary to prediction 2 and appears to be due to the increased aggressiveness of the capacity constrained agents in the uniform-price auctions. Panel B of Table 3 indicates that the correlation between revenue and the allocation received by capacity constrained bidders is -0.77 ($p = 0.03$).

Finally, in the Nash equilibria examined in Section 2, the discriminatory auctions have higher equilibrium revenue than the coalition-proof equilibrium revenue in the uniform price auctions, with both types of capacity constraints (prediction 1). Conditional on a lack of collusion the actual bidding is consistent with this prediction. Specifically, the average revenue of 494 in the discriminatory auctions with symmetric capacity constraints is significantly higher than the average revenue of 475.4 in the uniform-price auctions with symmetric capacity constraints ($p = 0.01$, in a one-tailed test). Similarly the average revenue of 499.8 for the discriminatory auctions with asymmetric bidding capacities is significantly higher than the average revenue of 488.8 for the uniform-price auctions with asymmetric bidding capacities ($p = 0.09$). As noted above, the discriminatory auction is more susceptible to collusion which explains why the unconditional average revenues are not consistent with prediction 1.

4.4 Allocations

[13] Each of these average revenues is significantly different from the revenue in the collusive cases (442).

In our asymmetric setting, three players had potential bidding capacities of 26 (unconstrained bidders) while two players had potential bidding capacities of 13 (capacity constrained bidders). The unique equilibrium in the discriminatory auction results in the following allocation: 6.5 units to the players that can submit 26 units and 3.25 units to the players that can submit 13 units. The actual allocation to high capacity bidders averaged 5.63 units. This is significantly less than the theoretical prediction (p=0.06) and is consistent with the finding that high capacity bidders bid, on average, for only 76.4% of their capacity. The uniform price mechanism supports multiple equilibria and each can be attained from several strategies. Hence, the uniform-price auction supports several allocations.

Table 4 describes the allocations according to the capacity level, the mechanism type and the location of the experiment. We find that on average under both auction formats, the unconstrained bidders receive larger allocations. Unconstrained bidders on average received higher allocations under the uniform-price mechanism than under the discriminatory mechanism (6.20 versus 5.63, p=0.14). This implies that the allocation between the two types of bidders is more symmetric in the discriminatory price mechanism than in the uniform-price mechanism.[14] We also find that under the discriminatory price mechanism the unconstrained bidders, on average, receive a lower allocation than the theory predicts while the capacity constrained bidders receive on average more. This reflects collusive agreements in which the bidders condition on their competitors' capacity constraints.

To further examine the symmetry of allocations of the good under the different mechanisms we compute the Herfindahl-Hirschman (HH) index of the allocations. The range of values for this index is 2000 if the allocation is perfectly symmetric to 10,000 if a single bidder

[14] A similar result is reported in SSZ (2004) who document that the discriminatory price mechanism leads to more symmetric allocations than the uniform-price mechanism. In their setting, the bidders always have symmetric bidding capacities.

receives the entire allocation. We perform an ANOVA analysis measuring the conditional average values of the index in the last auction of each session controlling again for the presence or absence of collusion, whether the auction is uniform-price or discriminatory, and whether the outcome is collusive or not. The evidence is again consistent with the hypothesis that the impact of asymmetric capacity constraints is due to the limitations they place on collusive behavior.

First, we see that, except for the case of the discriminatory auction with symmetric bidding capacities, the average HH index is significantly higher in the non-collusive outcomes than it is in the collusive outcomes for all types of auctions. In the uniform-price auctions, with symmetric capacities the average HH index for the non-collusive auctions (5090) is significantly ($p < 0.01$) higher than the average for the collusive auctions (2000). In the uniform-price auctions with asymmetric capacities the average HH index for the non-collusive auctions (3576) is significantly ($p = 0.05$) higher than the average for the collusive auctions (2444). In the discriminatory auctions with asymmetric capacities the average HH index for the non-collusive auctions (4095) is significantly ($p = 0.05$) higher than the average for the collusive auctions (2076). In the discriminatory auctions with symmetric capacity constraints nearly all of the outcomes were either perfectly collusive or competitive, both of which imply perfectly symmetric allocations. The symmetry of allocations is not significantly different in the collusive and non-collusive auctions for this reason. These results suggest that symmetry of allocations across bidders is an important component of collusive behavior.

Secondly, as with revenue, the impact of asymmetric capacity constraints on allocations is significant only for the discriminatory auctions. Controlling for a lack of collusion, the average level of the HH index in the discriminatory asymmetric capacity auctions (4095) is significantly ($p = 0.04$) greater than the average level of the HH index (2044) for the discriminatory auctions with symmetric capacity constraints. On the other hand, again controlling for a lack of collusion, the average level of the HH index for the uniform-price auctions with asymmetric capacity constraints (3576) is actually lower than the average level of the HH index (5090) for the uniform-price auctions with symmetric capacity constraints (although the difference is not significant).

4.5 Bidding aggressiveness and profits as a function of capacity constraints

Under both auction mechanisms, low capacity bidders bid more aggressively (submit demand curves with higher quantity-weighted average prices) than do high capacity bidders. The difference however is small, and is not statistically significant. We also calculate aggressiveness with the first six units of each bidder's demand curve, since in many cases these will be most relevant in determining allocations and profits. The results are essentially unchanged.

On average, under both auction mechanisms, unconstrained bidders earned higher profits than capacity constrained bidders. Profits of capacity constrained bidders were 63.9% of those of unconstrained bidders under the uniform-price auction and 80.4% of the profits of unconstrained bidders under the discriminatory auction. Treating each session average as an observation, however, neither difference is significant due to the large variation across sessions.

5. Conclusion

We use an experimental approach to analyze the impact of asymmetry in capacity constraints on bidding in a divisible good auction. The experimental economics laboratory is a natural setting in which to study this issue due to the complicated nature of the auction game when the good is divisible and the play is repeated. We show that asymmetry in capacity constraints plays an important role in inhibiting collusion and promoting competitive outcomes. This effect appears to be related to the increased difficulty of coordination when there are fundamental differences between bidders.

Asymmetry in capacity constraints plays a more important role in the discriminatory auction than in the uniform-price auction due to the greater susceptibility of the discriminatory auction to collusion. We show that asymmetry in capacity constraints increases the average revenue in both types of auctions and that this increase in revenue is derived from a reduction in collusive activity on the part of the bidders. We also find that while capacity constrained bidders bid more aggressively their constraint is effective in the sense that they receive, on average, a lower allocation and earn lower profits than their unconstrained counterparts.

The results regarding the level of collusion and the revenue obtained under each mechanism across different bidding capacity regimes suggest that the optimal auction format may depend heavily on factors specific to a particular auction setting. Universal and absolute revenue rankings may not exist when bidders are allowed to submit demand curves in multiple-unit auctions.

While one is always cautious in drawing policy implications from an experiment in a stylized setting, we interpret our results as suggesting that existing auction policies which set demand limits, and so tend to make bidders similar with respect to bidding capacity, may be counter productive from this perspective.[15] Instead, policies designed to attract more bidders to the auction may be more productive in encouraging competition and raising the seller's revenue.

[15] There are other reasons these limits exist. See the discussion in Jagedeesh (1993) of the Salomon squeeze in the U.S. Treasury auctions.

References

Armantier, O., and E. Sbaï, 2003, "Estimation and Comparison of Treasury Auction Formats when Bidders are Asymmetric," working paper SUNY Stony Brook.

Ausubel, L., and P. Cramton, 2002, "Demand Reduction and Inefficiency in Multi-Unit Auctions," University of Maryland working paper.

Back, K., and J. F. Zender, 1993, "Auctions of Divisible Goods: On the Rationale for the Treasury Experiment," *Review of Financial Studies*, **6**, 733-764.

Back, K., and J. F. Zender, 2001, "Auctions of Divisible Goods with Endogenous Supply", *Economics Letters*, **73**, 29-34

Bartolini, L., and C. Cottarelli, 1997, "Designing Effective Auctions for Treasury Securities", FEDERAL RESERVE BANK OF NEW YORK. *Current Issues in Economics and Finance*, **9**.

Benoit, J. P., and V. Krishna, 2001, "Multiple-Object Auctions with Budget Constrained Bidders", *Review of Economic Studies*, **68(1):** 155-79.

Friedman, M., 1960, A Program for Monetary Stability, Fordham University Press, New York.

Garbade D.,K and J.F Ingber, 2005, "The Treasury Auction Process: Objectives, Structure, and Recent Adaptations", FEDERAL RESERVE BANK OF NEW YORK. *Current Issues in Economics and Finance,* **11(2)**

Goswami, G., T. Noe, and M. Rebello, 1996, "Collusion in Uniform-Price Auctions: Experimental Evidence and Implications for Treasury Auctions," *Review of Financial Studies*, **9**, 757-785.

Jegadeesh, N., 1993, "Treasury Auctions Bids and the Salomon Squeeze," *Journal of Finance*, 48, 1403 – 1420.

Klemperer, P., 2002, "What Really Matters in Auction Design," *Journal of Economic Perspectives*, **16(1),** 169-189.

Kremer I. and K. Nyborg, 2002, "Divisible good auctions: The role of allocation rules", forthcoming *Rand Journal of Economics.*

Kremer I. and K. Nyborg, 2003, "Underpricing and Market Power in Uniform Price Auctions", forthcoming *Review of Financial Studies*

Pitchik, C and A. Schotter, 1988, "Perfect Equilibria in Budget-Constrained Sequential Auctions: An Experimental Study" *Rand Journal of Economics*. **19(3):** 363-88

Rey P., 2003, "Toward a Theory of Competition Policy" in Advances in Economics and Econometrics, Theory and Applications, Eighth World Congress, Volume 2
Edited by Mathias Dewatripont, Lars Peter Hansen, Stephen J. Turnovsky, Cambridge University Press

Sade, O., Schnitzlein, C. and J. F. Zender, 2004, "Competition and Cooperation in Divisible Good Auctions: An Experimental Examination", forthcoming *Review of Financial Studies*.

Wang, J. J. D., and J. F. Zender, 2002, "Auctioning Divisible Goods," *Economic Theory*, 19, 673-705.

Wilson, R., 1979, "Auctions of Shares," *Quarterly Journal of Economics*, **93**, 675-698.

Table 1

Experimental Sessions and Summary Statistics of the Asymmetric Setting

The table lists the experimental sessions conducted according to the date, mechanism, and the location and reports average prices and revenues. Bidding is permitted at four prices (20, 19, 18, 17) and it is common knowledge that the resale value at the end of each auction is 20. In sessions with asymmetric bidding capacities, three bidders may bid for up to 26 units while the remaining bidders are restricted to 13 units. When bidding capacities are symmetric, all five bidders may bid for up to 26 units.

Panel A. Uniform-Price Mechanism

Date	**Mechanism, Bidding Capacities, and Location**	Mean Stop-out Price	Mean Price	Mean Revenue
	Uniform-Price Symmetric Bidding Capacities			
10/7/2004	Hebrew University (Israel)	17.0	17.0	442
10/17/2004	Hebrew University (Israel)	17.5	17.5	455
11/6/2004	Hebrew University (Israel)	18.2	18.2	474
11/15/2004	Hebrew University (Israel)	19.0	19.0	494
9/16/2004	University of Central Florida (USA)	18.3	18.3	475
9/23/2004	University of Central Florida (USA)	19.5	19.5	507
9/30/2004	University of Central Florida (USA)	17.0	17.0	442
11/3/2004	University of Central Florida (USA)	18.9	18.9	490
	Mechanism Averages	**18.2**	**18.2**	**472.4**
	Uniform-Price Asymmetric Bidding Capacities			
11/19/2003	Hebrew University (Israel)	17.0	17.0	442
4/5/2004	Hebrew University (Israel)	17.7	17.7	461
1/4/2005	Hebrew University (Israel)	19.5	19.5	507
12/28/2004	Hebrew University (Israel)	18.1	18.1	472
2/18/2004	University of Central Florida (USA)	19.3	19.3	501
2/23/2004	University of Central Florida (USA)	17.9	17.9	466
3/3/2004	University of Central Florida (USA)	18.1	18.1	470
1/13/2005	University of Central Florida (USA)	18.1	18.1	470
	Mechanism Averages	**18.2**	**18.2**	**473.6**

Panel B. Discriminatory Price Mechanism

Date	Mechanism, Bidding Capacities, and Location	Mean Stop-out Price	Mean Price	Mean Revenue
	Discriminatory-Price Symmetric Bidding Capacities			
10/1/2004	Hebrew University (Israel)	17.7	17.7	461
10/10/04	Hebrew University (Israel)	17.0	17.0	442
10/24/04	Hebrew University (Israel)	19.0	19.0	494
11/29/04	Hebrew University (Israel)	17.0	17.0	442
9/15/2004	University of Central Florida (USA)	18.8	18.8	490
9/22/2004	University of Central Florida (USA)	17.9	18.1	471
9/29/2004	University of Central Florida (USA)	17.0	17.0	442
10/13/2004	University of Central Florida (USA)	18.7	18.9	491
	Mechanism Averages	**17.9**	**18.0**	**466.6**
	Discriminatory-Price Asymmetric Bidding Capacities			
11/19/2003	Hebrew University (Israel)	17.0	17.1	444
12/12/2003	Hebrew University (Israel)	18.7	18.9	492
12/14/2003	Hebrew University (Israel)	17.9	18.3	475
12/14/2003	Hebrew University (Israel)	18.9	19.2	499
2/11/2004	University of Central Florida (USA)	17.5	17.6	458
2/16/2004	University of Central Florida (USA)	18.9	19.0	494
3/4/2004	University of Central Florida (USA)	18.4	18.6	484
11/4/2004	University of Central Florida (USA)	17.0	17.0	442
	Mechanism Averages	**18.0**	**18.2**	**473.5**

Table 2
Evolution of Revenue – Asymmetry in Capacity

This table reports the evolution of revenue within experimental sessions and the standard deviation of revenue for each session.

Panel A: Uniform-Price Asymmetric Bidding Capacities

Date	Location	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Mean	Std Dev
2/18/2004	UCF	468	520	520	468	520	520	494	494	494	520	494	494	494	520	**501.4**	**18.9**
2/23/2004	UCF	494	468	468	494	442	442	520	520	442	442	442	468	442	442	**466.1**	**29.7**
3/3/2004	UCF	520	494	494	494	468	468	494	442	468	442	442	442	442	468	**469.9**	**25.9**
1/13/2005	UCF	442	468	520	494	494	520	520	442	442	442	442	442	442	468	**469.9**	**33.0**
1/4/2005	ISRAEL	494	494	494	520	520	520	494	494	494	520	494	520	520	520	**507.0**	**13.5**
12/28/2004	ISRAEL	468	520	468	442	520	494	494	494	468	442	442	442	442	468	**471.7**	**28.6**
11/19/2003	ISRAEL	442	442	442	442	442	442	442	442	442	442	442	442	442	442	**442.0**	**0.0**
4/5/2004	ISRAEL	494	468	494	494	520	442	442	442	442	442	442	442	442	442	**460.6**	**27.8**
	Mean	**477.8**	**484.3**	**487.5**	**481.0**	**490.8**	**481.0**	**487.5**	**471.3**	**461.5**	**461.5**	**455.0**	**461.5**	**458.3**	**471.3**	**473.6**	**22.2**
	Std Dev	**27.6**	**27.6**	**26.9**	**27.8**	**35.3**	**36.8**	**30.3**	**32.4**	**23.0**	**36.1**	**24.1**	**30.3**	**30.9**	**32.4**		

Panel B: Uniform-Price Symmetric Bidding Capacities

Date	Location	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Mean	Std Dev
9/16/2004	UCF	468	468	468	468	468	468	468	468	468	468	494	494	520	468	**475.4**	**15.9**
9/23/2004	UCF	468	520	520	520	494	520	494	520	494	520	520	520	494	494	**507.0**	**16.9**
9/30/2004	UCF	442	442	442	442	442	442	442	442	442	442	442	442	442	442	**442.0**	**0.0**
11/3/2004	UCF	494	494	494	520	494	494	494	494	494	494	494	468	468	468	**490.3**	**13.9**
10/7/2004	ISRAEL	442	442	442	442	442	442	442	442	442	442	442	442	442	442	**442.0**	**0.0**
10/17/2004	ISRAEL	442	442	494	442	468	442	468	442	442	520	442	442	442	442	**455.0**	**24.5**
11/15/2004	ISRAEL	494	520	468	494	494	494	494	494	468	494	494	520	494	494	**494.0**	**14.4**
11/6/2004	ISRAEL	442	494	494	494	468	494	520	520	442	442	442	442	468	468	**473.6**	**29.2**
	Mean	**461.5**	**477.8**	**477.8**	**477.8**	**471.3**	**474.5**	**477.8**	**477.8**	**461.5**	**477.8**	**471.3**	**471.3**	**471.3**	**464.8**	**472.4**	**14.3**
	Std Dev	**23.0**	**33.9**	**27.6**	**33.9**	**21.7**	**30.3**	**27.6**	**33.9**	**23.0**	**33.9**	**32.4**	**35.3**	**29.3**	**21.7**		

Panel C: Discriminatory-Price Asymmetric Bidding Capacities

Date	Location	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Mean	Std Dev
2/11/2004	UCF	463	481	494	442	442	442	446	494	494	442	442	442	442	442	**457.7**	**22.6**
2/16/2004	UCF	495	500	468	494	497	496	497	499	494	504	494	468	494	520	**494.3**	**13.0**
3/4/2004	UCF	495	495	442	468	468	473	491	470	483	497	494	494	507	496	**483.8**	**17.4**
11/4/2004	UCF	442	442	442	442	442	442	442	442	442	442	442	442	442	442	**442.0**	**0.0**
11/19/2003	ISRAEL	442	444	442	472	442	442	442	442	442	442	442	442	442	442	**444.3**	**8.0**
12/12/2003	ISRAEL	459	492	494	472	494	494	507	507	494	494	494	494	494	494	**491.6**	**12.3**
12/14/2003	ISRAEL	468	468	494	496	498	497	497	462	462	461	453	455	462	477	**475.0**	**17.5**
12/14/2003	ISRAEL	493	492	494	483	507	468	494	494	494	504	512	520	520	512	**499.1**	**14.4**
	Mean	**469.6**	**476.8**	**471.3**	**471.1**	**473.8**	**469.3**	**477.0**	**476.3**	**475.6**	**473.3**	**471.6**	**469.6**	**475.4**	**478.1**	473.5	13.2
	Std Dev	**22.4**	**23.0**	**25.8**	**20.7**	**28.5**	**24.9**	**28.3**	**25.9**	**23.5**	**29.2**	**29.5**	**29.8**	**32.1**	**32.5**		

Panel D: Discriminatory-Price Symmetric Bidding Capacities

Date	Location	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Mean	Std Dev
9/15/2004	UCF	494	494	494	500	500	500	447	468	494	468	520	499	494	494	**490.4**	**18.0**
9/22/2004	UCF	455	492	494	442	494	442	464	494	495	494	502	442	442	442	**471.0**	**25.7**
9/29/2004	UCF	442	442	442	442	442	442	442	442	442	442	442	442	442	442	**442.0**	**0.0**
10/13/2004	UCF	479	494	486	494	494	494	494	494	478	488	494	494	494	494	**490.8**	**5.8**
11/1/2004	ISRAEL	494	494	494	494	494	442	442	442	442	442	442	442	442	442	**460.6**	**25.9**
10/10/2004	ISRAEL	442	442	442	442	442	442	442	442	442	442	442	442	442	442	**442.0**	**0.0**
10/24/2004	ISRAEL	494	494	494	494	494	494	494	494	494	494	494	494	494	494	**494.0**	**0.0**
11/29/2004	ISRAEL	442	442	442	442	442	442	442	442	442	442	442	442	442	442	**442.0**	**0.0**
	Mean	**467.8**	**474.3**	**473.5**	**468.8**	**475.3**	**462.3**	**458.4**	**464.8**	**466.1**	**464.0**	**472.3**	**462.1**	**461.5**	**461.5**	**466.6**	**9.4**
	Std Dev	**24.9**	**26.7**	**26.2**	**28.7**	**27.6**	**28.0**	**23.2**	**25.8**	**26.3**	**24.9**	**33.3**	**27.8**	**26.9**	**26.9**		

Table 3
Comparison between the Symmetric and Asymmetric Settings

The table compares the revenue and price results under asymmetric setting to the symmetric setting. In both treatments, bidding is permitted at four prices (20, 19, 18, 17) and it is common knowledge that the resale value at the end of each auction is 20. In the asymmetric setting, two of the bidders can bid up to 13 units while three can bid for 26 units. In the symmetric setting all the bidders can bid for 26 units. The capacity constrained bidders' profit share is the ratio of the profits of capacity-constrained bidders' to total profits.

Panel A. Discriminatory Mechanism

	Asymmetric Capacities	**Symmetric Capacities**	**P-value of difference**
Collusive Auctions (Rev=442)	35/112	56/112	P<0.01 Fisher exact
Collusive Sessions	1/8	3/8	0.57 Fisher exact
Collusive Auctions (Last auction)	3/8	5/8	0.62 Fisher exact
Last auction: revenue consistent with collusive outcome or Nash-equilibrium	4/8	8/8	0.08 Fisher exact
Auctions with bids at 20	25/112	5/112	p<0.01
Bids as % of capacity	81.2%	99.8%	p<0.01
Revenue	473.5 (N=112)	466.6 (N=112)	0.05 t-test
Revenue (Session means) N=8	473.5	466.6	0.56 t-test (N=8) 0.55 rand. test
Revenue (last auction) N=8	478.1	461.5	0.28 t-stat 0.26 rand. Test
Variance (average of session averages)	214.2	210.8	0.97 rand. Test
Variance across sessions (last auction)	1055.6	724.3	0.32 F-test for variance
Correlation: Capacity constrained bidders' allocation with auctioneer's revenue	-0.45 (p-value=0.20)		

Panel B. Uniform-Price Mechanism

	Asymmetric Capacities	Symmetric Capacities	P-value of difference
Collusive Auctions (Rev=442)	47/112	43/112	0.68 Fisher exact
Collusive Sessions	1/8	1/8	1.00 Fisher exact
Collusive Auctions (Last auction)	3/8	3/8	1.00 Fisher exact
Last auction: revenue consistent with either the collusive outcome or Nash-equilibrium	3/8	3/8	1.00 Fisher Exact
Bids as % of capacity	88.1%	99.9%	p<0.01
Revenue	473.6 (N=112)	472.4 (N=112)	0.77 t-stat
Revenue (Session means) N=8	473.6	472.4	0.92 t-stat 0.91 rand. Test
Revenue (last auction)	471.3	464.8	0.65 t-stat 0.81 rand. Test
Variance (average of session averages)	596.1	298.5	0.10 rand. Test
Variance across sessions (last auction)	1050.2	470.8	0.16 F-test for variance
Correlation: Capacity constrained bidders' allocation with auctioneer's revenue	-0.77 (p-value=0.03)		

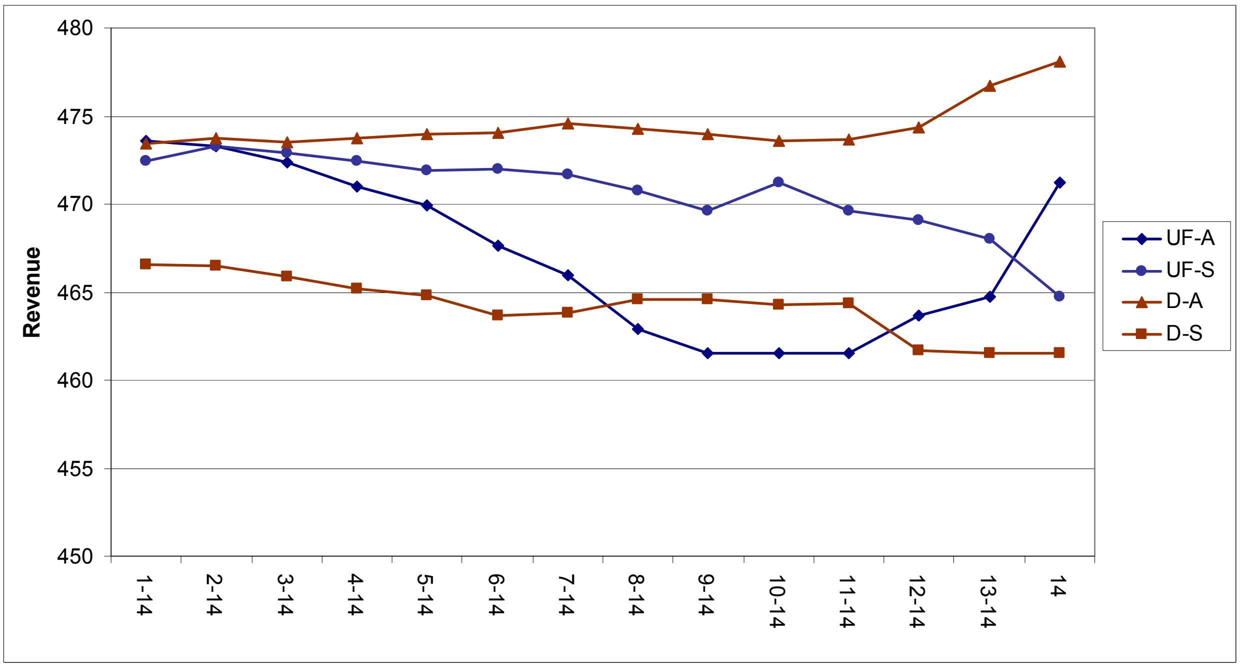
Table 4
Allocations according to the Mechanism, Type of Capacity Constraint and the Location of the Experiment.

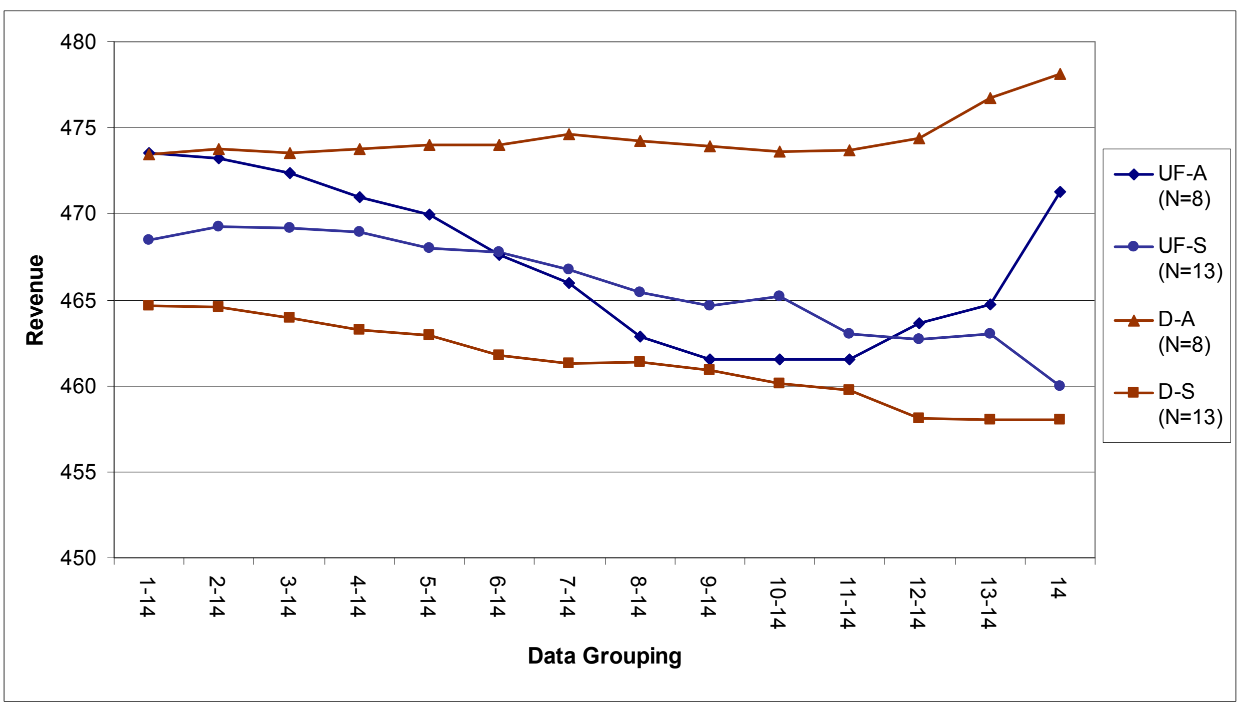
The capacity constrained bidders' profit share is the ratio of the profits of capacity-constrained bidders' to total profits.

Date	**Mechanism, and Location**	Mean Allocation		Mean Profit		Capacity constrained bidders' profit share
	Discriminatory-Price	Bidding Capacity=26	Bidding Capacity=13	Bidding Capacity=26	Bidding Capacity=13	
11/19/03	Hebrew University (Israel)	5.04	5.44	211.0	213.50	0.50
12/12/03	Hebrew University (Israel)	6.05	3.92	98.15	51.28	0.34
12/14/03	Hebrew University (Israel)	5.38	4.93	58.08	59.38	0.51
12/14/03	Hebrew University (Israel)	4.81	5.79	121.76	132.36	0.52
2/11/04	University of Central Florida (USA)	5.55	4.68	187.45	154.82	0.45
2/16/04	University of Central Florida (USA)	7.05	2.42	100.85	28.72	0.22
3/4/24	University of Central Florida (USA)	5.98	4.03	116.33	79.00	0.40
11/4/04	University of Central Florida (USA)	5.20	5.20	218.4	218.4	0.50
	Mechanism Averages	**5.63**	**4.55**	**139.0**	**117.2**	**0.43**
	Uniform-Price					
11/19/03	Hebrew University (Israel)	5.23	5.18	219.6	217.6	0.50
4/5/04	Hebrew University (Israel)	6.63	3.05	209.48	101.79	0.33
1/4/2005	Hebrew University (Israel)	7.62	1.57	48.72	17.92	0.27
12/28/04	Hebrew University (Israel)	5.81	4.29	150.61	112.09	0.43
2/18/04	University of Central Florida (USA)	6.52	3.22	67.57	28.66	0.30
2/23/04	University of Central Florida (USA)	5.92	4.12	161.48	134.79	0.45
3/3/04	University of Central Florida (USA)	6.46	3.31	170.88	94.68	0.36
1/13/05	University of Central Florida (USA)	5.5	4.75	158.68	112.98	0.42
	Mechanism Averages	**6.2**	**3.7**	**148.4**	**102.6**	**0.38**

Figure 1
Experience Effects

The following figure shows the influence of experience on auctioneer's revenue by comparing results from later auctions with global averages. The first data grouping is the global average for all 14 auctions under each mechanism, the second data grouping is the average for auctions 2 through 14, the third is the average for auctions 3 through 14 and so on. The second figure includes sessions with symmetric capacity constraints from SSZ (2004).





Appendix A – Instructions

Auction-2 (UF)

This is an experiment in economic decision-making. The experiment consists of several rounds. At the end of each round your payoff for that round will be calculated. At the end of the experiment, your payoff from each round will be added up and this sum will determine your payoff for the experiment. Your payoff will be made with funds provided through grants by various institutions. Please feel free to earn as much of this money as possible. Everything contained in these instructions and everything you hear in this session is an accurate representation of this experiment. Be sure to ask any questions that you may have during this instruction period, and ask for assistance, if needed, at any time. All subjects receive the same instructions.

There are four parts in today's experiment:

1. These instructions
2. The trading game consisting of a random number of auctions
3. A questionnaire
4. The (private) payment of earnings

THE TRADING GAME OVERVIEW

In this experiment you will be required to bid for units of a good which we will call widgets. There will be 26 widgets available to all players. The resale value of each widget at the end of the auction is Fr. 20. You will be submitting a schedule of bids. This schedule indicates the number of widgets you are willing to buy at a given price level. The possible price levels will be Fr. 17, Fr. 18, Fr. 19, and Fr. 20. Once all schedules have been submitted, the computer will assign widgets to players submitting the highest bids until the available supply of 26 widgets is exhausted. All the players will pay the same price (clearing price) for each widget he/she is allocated. At the end of each auction your cash balance will increase by 20 francs for each widget you hold, so you will earn profits on each widget you purchase at a clearing price less than 20 francs. There will be multiple auctions. The exact number will be randomly chosen. At the end of the experiment your balance in francs will be converted to cash (1 franc = $0.10), and you will be paid that amount in cash. Only you and the assistant that pays you will learn your earnings from today's session.

DETAILS

There will be **26** widgets available for sale. Your resale value for each widget is **20 francs.** (This means that after the auction your balance will increase by 20 francs for each widget that you hold, less what you paid for each widget). Prior to each auction, you will be required to submit via computer a schedule of bids. This schedule indicates the number of widgets you are willing to buy (including zero) at each possible price level. The possible price levels will be Fr. 17, Fr. 18, Fr. 19, and Fr. 20 and the sum of all of your bids may not exceed the number of units indicated at your computer terminal. This number may be different for different participants. Once each participant has submitted his/her schedule of bids, the computer will calculate the highest price at which all 26 widgets can be sold and will allocate widgets to players that submit bids that are equal to or higher than this price. **The price paid for each widget will be equal to the clearing price. The market-clearing price will be the highest price at which the total demand for widgets summed across all bidders is equal to 26.**

There will be several rounds of bidding. Only you will know your actual bids, allocations, and payoffs in each round, however all the participants will know the eventual clearing price of each round. You have the option to change your bids from one auction to another. The computer will end the experiment after some random number of rounds of bidding. Only the computer knows this number of rounds.

The Calculation of Your Payoffs

Your payoffs from each auction will be equal to the difference between your resale value for each widget (Fr.20) and the clearing price times the number of widgets you are allocated. At the end of each round your payoffs will be calculated. At the end of each experiment, the payoffs of each round will be added up. The sum of your round by round payoffs will determine your payoff from the experiment.

After the experiment has ended, your cumulative dollar payoff will be determined using the following formula: $ Payoffs = Balance in Francs * 0.10

Each participant will learn only his/her own final cash payment.

COMMUNICATION RULES

Before and after each round you may discuss strategies with the other players for about 1 minute. However you are required to remain seated behind your computer screen at all times. You are explicitly **not allowed to**:

1) Make physical threats of any kind or verbally abuse other players
2) Agree to share profits after the experiment
3) Look at the computer screen of any other player
4) Ask other participants how much they have earned when the experiment has ended.

The following examples are for illustrative purposes only. They are not intended to be suggested as "best" strategies and simply demonstrate the implications of a possible set of actions.

EXAMPLE 1

Consider a game with 5 bidders: A, B, C, D and E. Suppose they submit the following schedules:

	Bidders					Demand	Cumulative Demand	Supply
PRICE	A	B	C	D	E			
20	11	0	5	0	0	16	16	26
19	5	0	3	2	0	10	26	26
18	5	0	8	6	18	37	63	26
17	5	26	10	18	8	67	130	26

The demand at each price is the sum of the demands of bidders A, B, C, D, and E. For example the demand at price 20 is equal to 11+0+5+0+0=16. The cumulative demand is equal to the total demand at that price and all higher prices. For example the cumulative demand at the price of 19 is 16 (Demand at 20) + 10 (Demand at 19) = 26. The market-clearing price is the highest price at which the cumulative demand equals the supply. In this case, the cumulative demand equals the supply at price equal 19.

The allocation and profit of the players is as follows:

PRICE	A	B	C	D	E
Allocation	16	0	8	2	0
Profit	16*(20-19)=**16**	**0**	8*(20-19)=**8**	2*(20-19)=**2**	**0**

Since the value of a widget for each player is 20, each player makes a profit for each unit that he/she bought at a price below 20. Since in this example the clearing price is 19, each participant makes a profit of 1 Fr. times the number of units he/she is allocated.

Note: At the end of each round, you will learn the total demand at each price and your own payoff, but you will not learn the bids or payoffs of any other participant.

EXAMPLE 2

The following example illustrates a case where cumulative demand does not exactly equal supply at any price, and shows how the widgets are allocated if this occurs.

	Bidders					**Demand**	**Cumulative Demand**	**Supply**
PRICE	**A**	**B**	**C**	**D**	**E**			
20	11	0	5	0	0	16	16	26
19	5	0	3	2	8	18	34	26
18	5	0	8	6	10	29	63	26
17	5	26	10	18	8	67	130	26

In this case, the market-clearing price is 19. Each player will be allocated his/her demand at price 20 and ***some*** of his/her demand at price = 19. Each player's allocation at 19 will depend on how large his/her bid was at that price: the larger the bid the larger the allocation as follows. Player A bids for 5 units at a price of 19. Since these 5 units represent 5/18 or 27.7% of demand at price=19, this player receives 27.7% of the number of widgets necessary to make cumulative demand at price = 19 equal 26 (the supply of widgets). Since 10 widgets will be allocated from bids at price = 19, player A receives 2.8 of these widgets (27.7% of 10 rounded to the tenth). Player A will also receive his/her entire bid of 10 at price = 20.

The allocation and profit of the players is as follows:

PRICE	**A**	**B**	**C**	**D**	**E**
Allocation	13.8	0	6.7	1.1	4.4
Profit	13.8*(20-19)=**13.8**	0	6.7*(20-19)=**6.7**	1.1*(20-19)=**1.1**	4.4*(20-19)=**4.4**

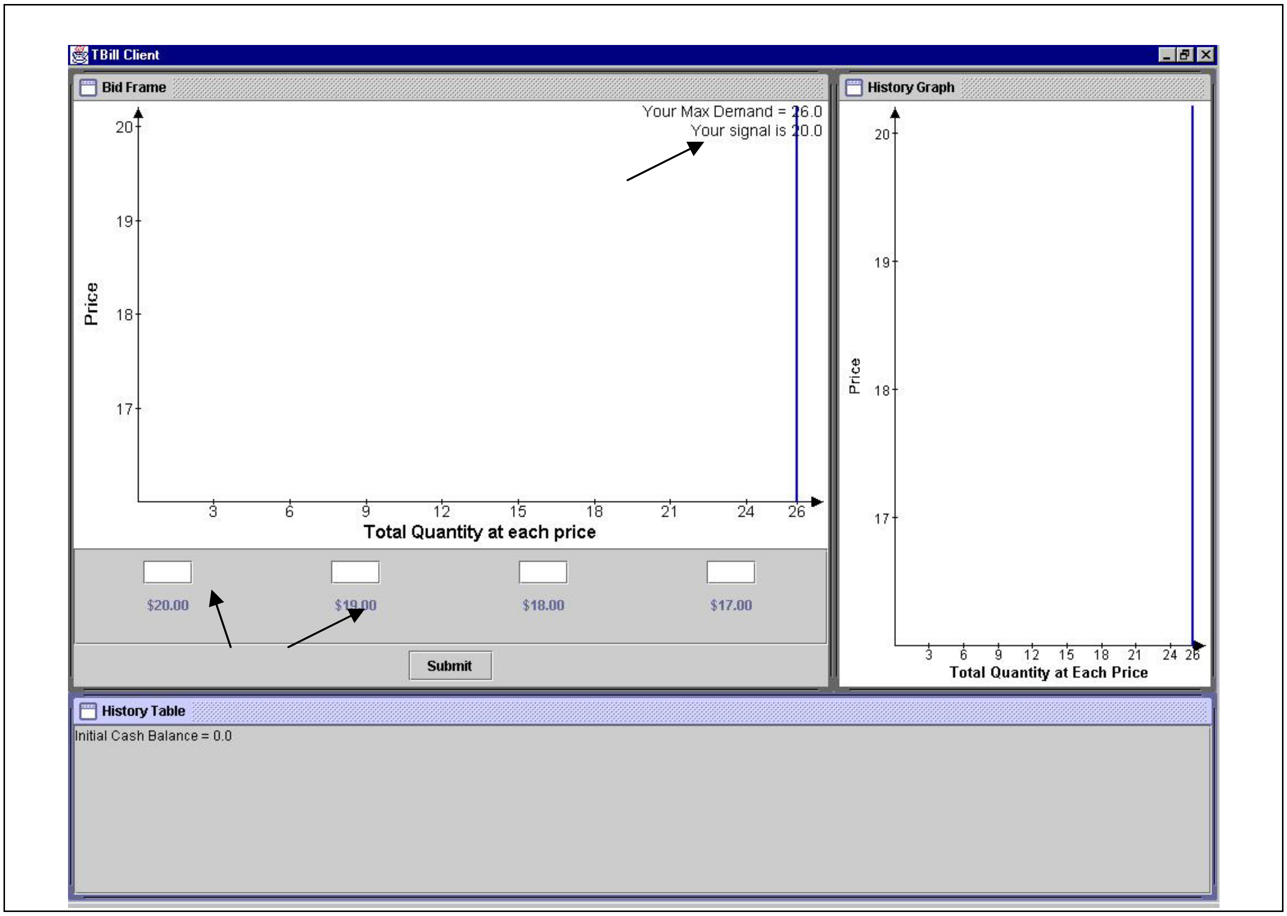
GRAPHICAL INTERFACE

The following figures are for illustrative purposes only. They are not intended to be suggested as "best" strategies and simply demonstrate the computer interface.

Figure 1 below shows the first screen you will see when the game starts. The computer screen is divided into three main areas: The "Bid Frame" (large area on the left-upper corner), the "History

Graph" (thin rectangular area on the right-upper corner) and the "History Table" (wide rectangular area on the bottom of the screen).

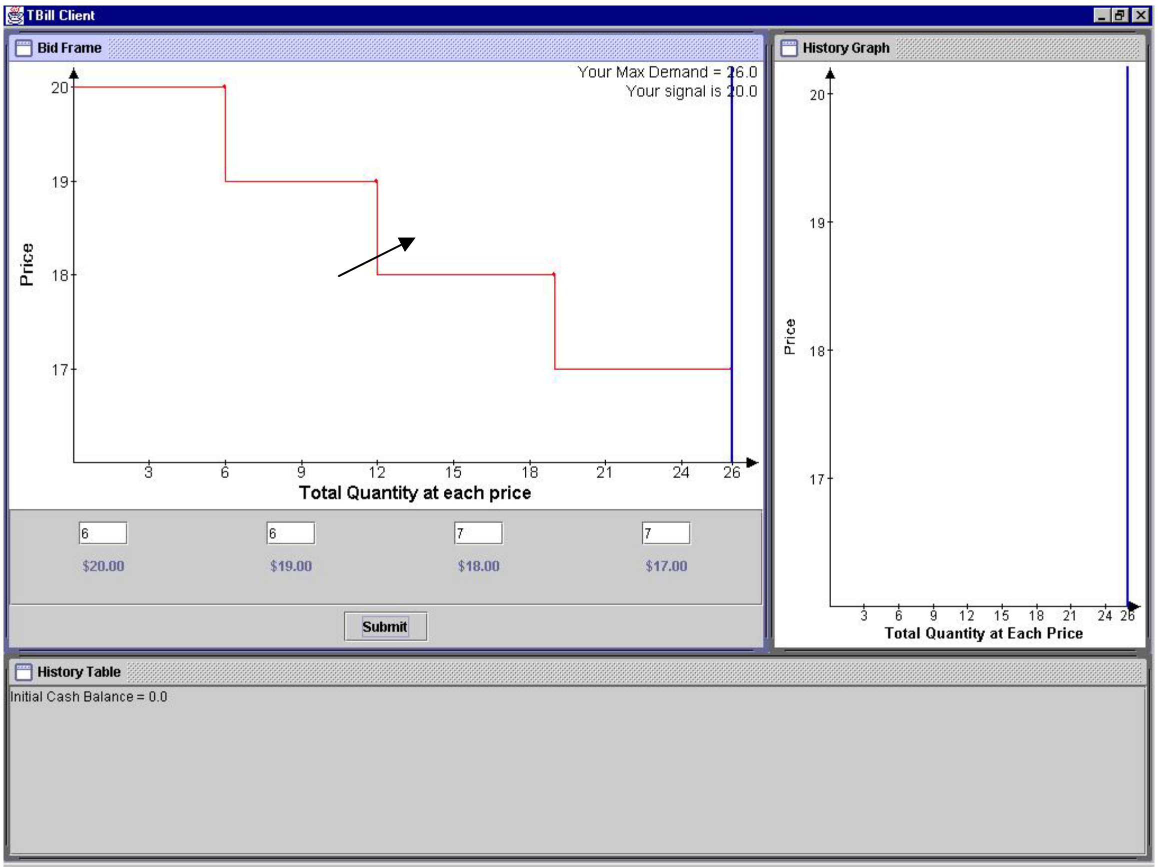
Figure I



The Bid Frame is the interface that allows you to input your bids at each price. Point "A" reminds you that your resale value for each widget will be Fr. 20. The white fields marked with "B" are where you will input your bids at the different prices. Notice that there is one field for each of the four allowed prices (that is Fr. 20, Fr. 19, Fr. 18 and Fr. 17). You must submit your bid at each respective price by typing a non-negative integer between 0 and 26. By default the computer positions your cursor in the Fr. 20 Field. You can move to the next field (Fr. 19) by pressing Tab on your keyboard or by clicking the field with the left button of your mouse.

Figure 2 below shows you how the process just described works. You will notice that as you input your demand schedule in the mentioned fields, the computer will draw a demand curve (demand graph). It will look like the downward sloping ladder graph marked with the letter "C" in Figure 2.

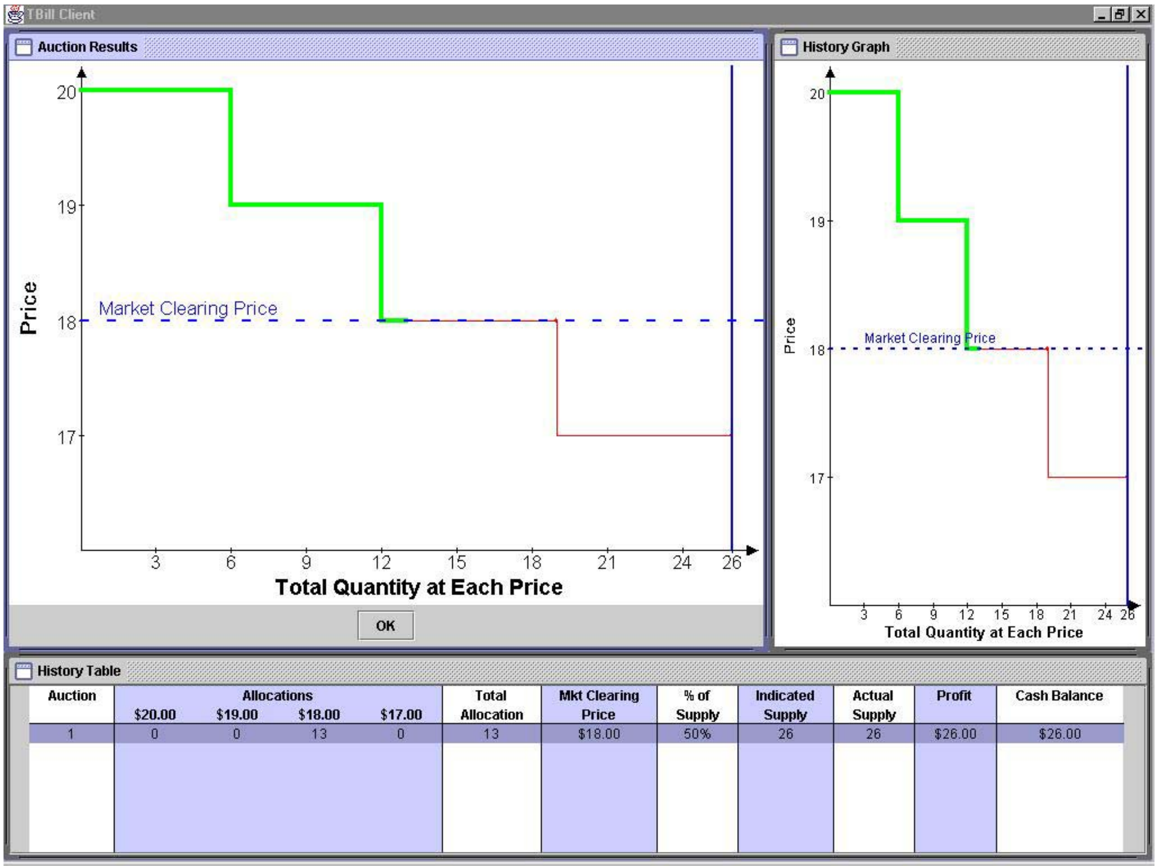
Figure 2



Note that this subject has a maximum demand of 26. Recall that different subjects may have different maximum demands. You are allowed to bid for less than this amount but not more. Once you are done inputting your bids, you submit them by clicking the "Submit" button on the screen with your mouse, or by "tabbing" into it with your keyboard and pressing the "Enter" key.

Once all subjects have submitted their bids, the results for the auction will be displayed. The results for the round just played will always be displayed in the same area as the "Bid Frame" (the large area on the right-upper corner of the screen). Notice however that the label of the Frame has changed to "Auction Results." Figure 3 shows the hypothetical results of the first round in our example. The "Auction Results" Window will show your original demand curve, the market clearing price with a dotted blue line, and will highlight with green the portion of your demand curve which was filled (in the case of our example 13 widgets).

Figure 3



The “History Table” gives you a numerical summary of the round’s results. It tells you the number of widgets you were allocated, the market-clearing price, the percentage of the overall supply of 26 widgets you received, your profits, and your cash balance.

Figure 4

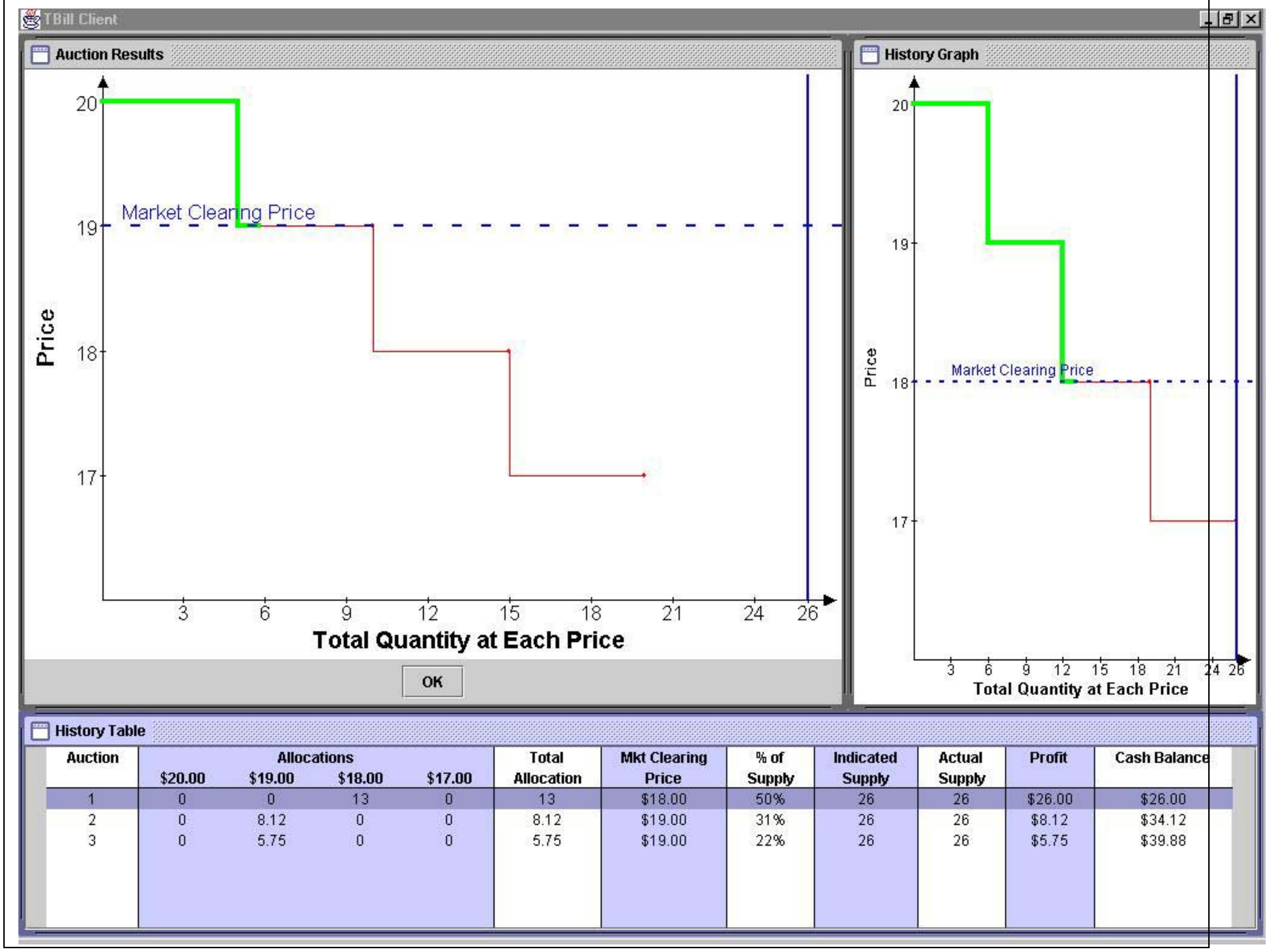


Figure 4 shows how the “History Graph” displays results from previous rounds. In this illustration, even though the game has already been played three rounds, the graph (upper right-hand corner) is showing the results of the first one. Please notice how the “Auction Results” Graph and the “History Graph” differ: the “Auction Results” Graph shows the results of the last round played (in this case round 3) and the “History Graph” is showing those of the first round.

At any point in the game you can recall the “History Graph” from past rounds by double clicking the desired round in the “History Table.” Notice in Figure 4 that although the “History Table” depicts the results of all the rounds played, the first round is highlighted. This means that the "History Graph" shown corresponds to that highlighted round. By default, after each round, the "History Table" will highlight the last round played, but you can change this at any point of the game by highlighting the desired round. When a game lasts for more rounds than the "History Table" can show in its limited area, a scroll bar will appear to the right of the table. This allows you to view the results from any period.

The computer will randomly determine the last auction. When this occurs a screen will be displayed that reports your initial cash balance (zero) and your profits at the end of the game. If you wish you may write down your profits, but you need not do so; the experimenter has this record and will pay you your exact earnings in private. Finally, press "Enter" with the keyboard (or click "Ok" using the mouse) and your screen will go blank. If you don't do this, the screen will automatically go blank after 60 seconds. Either way, do not get up from your seat until instructed to do so.

Quiz

1. Suppose you bid for 1 widget at a price of Fr. 20, 10 widgets at a price of Fr. 19, 2 widgets at a price of Fr. 18, and 13 widgets at a price of Fr. 17. Suppose you receive you receive all the widgets you bid for at a price greater than or equal to Fr.18, and none of the widgets you bid for at a price of Fr. 17. What is your profit in Francs for the period?

2. Assume the following set of bids.

	Bidders					**Demand**	**Cumulative Demand**	**Supply**
PRICE	**A**	**B**	**C**	**D**	**E**			
20	1	0	5	8	1	15	15	26
19	2	0	5	0	1	8	23	26
18	10	0	5	3	12	30	53	26
17	13	26	11	15	12	77	130	26

a. What is the market-clearing price?

b. What is the profit of player B?

c. What is the profit of player D?